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Simultaneous Measurements of Thermal Conductivity and Thermal Diffusivity of Liquids Under Microgravity Conditions[1]

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A transient short-hot-wire technique is proposed and used to measure the thermal conductivity and thermal diffusivity of liquids simultaneously. The method is based on the numerical evaluation of unsteady heat conduction from a wire with the same length diameter ratio and boundary conditions as those in the experiments. To confirm the applicability and accuracy of this method, measurements were made for five sample liquids with known thermophysical properties and were performed under both normal gravity and microgravity conditions. The results reveal that the present method determines both the thermal conductivity and the diffusivity within 2 and 5%, respectively. The microgravity experiments clearly indicate that even under normal gravity conditions, natural-convection effects are negligible for at least 1 s after the start of heating. This method would be particularly suitable for a valuable and expensive liquid, and has a potential for application to electrically conducting and/or corrosive liquids when the probe is effectively coated with an insulating and anticorrosive material.

KEY WORDS: microgravity; natural convection; thermal conductivity; thermal diffusivity; transient short-hot-wire technique.

1. INTRODUCTION

Many methods have been proposed to measure the thermal conductivity of liquids, and the transient hot-wire method with long wires is the most

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promising [1]. This method is, however, rather difficult to apply for electrically conducting and highly corrosive fluids, such as molten carbonate salts, because the wirings must all be insulated with anticorrosive and nonconducting materials. These difficulties could be much reduced if we could develop a transient hot-wire method with much shorter wires than the conventional ones. From this point of view, the authors [2] proposed a new method which uses a hot wire as short as about 10 mm. The time evolution of the wire temperature is measured, and by comparison with the numerical simulations, the thermal conductivity and thermal diffusivity of fluids are determined simultaneously.

In this paper, the numerical analysis, on which the present method is based and the experimental arrangement are described. Also, results of the simultaneous measurement of the thermal conductivity and thermal diffusivity of five sample liquids—pure water, ethanol, methanol, acetone, and toluene—are presented. These measurements were carried out under both normal and microgravity conditions at room temperature and atmospheric pressure. The results are compared with relevant reference values. Further, the effect of natural convection on such a short-hot-wire probe is also discussed.

2. NUMERICAL ANALYSIS

2.1. Physical Model

Figure 1 shows the physical model and the coordinate system. The sample liquid is filled in a cylindrical vessel of radius r_0, and a short hot

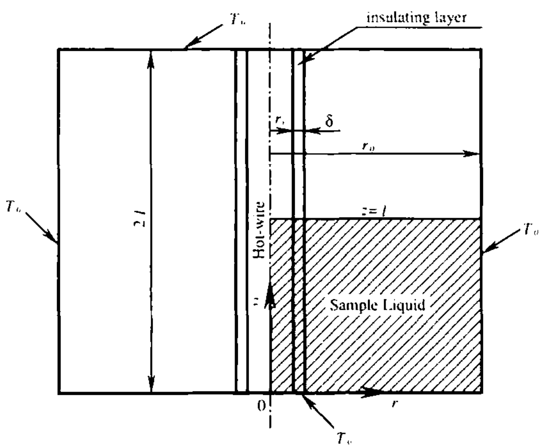


Fig. 1. Physical model and coordinate system.

wire of radius r_i and length $2l$ is located on the center axis and is supported by lead wire at each end. The hot wire is covered by an insulating material of thickness δ. The two-dimensional heat-conduction equations for this three-layer model are numerically solved under the following assumptions: (1) the heat generation rate per unit volume and time is uniform and constant; (2) the temperature of the lead wires is kept at its initial temperature T_0 during the heating process; and (3) the temperature distribution in the vessel is symmetric with respect to the z-axis and the plane normal to it at $z = l$. Therefore, the hatched region in Fig. 1 is taken as the solution domain.

Nondimensional basic equations are written as follows.

For the hot wire,

$$\frac{\partial \theta_{sh}}{\partial Fo} = \frac{1}{R_{d1}}\left(\frac{\partial^2 \theta_{sh}}{\partial R^2} + \frac{1}{R}\frac{\partial \theta_{sh}}{\partial R} + \frac{\partial^2 \theta_{sh}}{\partial Z^2}\right) + \frac{R_{c1}}{R_{d1}} \tag{1}$$

For the insulating layer and liquid layer,

$$\frac{\partial \theta_{si}}{\partial Fo} = \frac{1}{R_{d2}}\left(\frac{\partial^2 \theta_{si}}{\partial R^2} + \frac{1}{R}\frac{\partial \theta_{si}}{\partial R} + \frac{\partial^2 \theta_{si}}{\partial Z^2}\right) \tag{2}$$

$$\frac{\partial \theta_f}{\partial Fo} = \frac{\partial^2 \theta_f}{\partial R^2} + \frac{1}{R}\frac{\partial \theta_f}{\partial R} + \frac{\partial^2 \theta_f}{\partial Z^2} \tag{3}$$

where θ, R, and Z are the dimensionless temperature and radial and longitudinal coordinates, respectively. Fo is the Fourier number. These are defined as

$$\theta = \frac{T - T_0}{q_v r_i^2/\lambda}, \qquad Fo = \frac{\alpha t}{r_i^2}, \qquad R = \frac{r}{r_i}, \qquad Z = \frac{z}{r_i} \tag{4}$$

The parameters R_{c1}, R_{d1}, etc., with those appearing in the boundary conditions are the thermal conductivity and diffusivity ratios of each layer, defined as

$$R_{c1} = \frac{\lambda}{\lambda_{sh}}, \qquad R_{c2} = \frac{\lambda_{si}}{\lambda_{sh}}, \qquad R_{c3} = \frac{\lambda}{\lambda_{si}}, \qquad R_{d1} = \frac{\alpha}{\alpha_{sh}}, \qquad R_{d2} = \frac{\alpha}{\alpha_{si}} \tag{5}$$

where λ and α are the thermal conductivity ($W \cdot m^{-1} \cdot K^{-1}$) and the thermal diffusivity ($m^2 \cdot s^{-1}$), and where the subscripts sh and si indicate short hot wire and insulating material, respectively.

These equations are solved by a finite-difference method with an implicit subsequent substitution scheme, under relevant initial and boundary conditions. A nonuniform grid arrangement is used and the node numbers are 101 and 201 in Z and R directions, respectively.

2.2. Numerical Results

Numerical analysis yields the dimensionless volume-averaged hot-wire temperature θ_v as a function of the dimensionless heating time Fo. The numerical error is confirmed to be within 0.2% for the range of Fo = 5–100, when the solution corresponding to an infinite-length wire is compared with that obtained analytically. The effect of the boundary conditions at $R = R_o$ is also examined to estimate the minimum radius of the vessel to achieve accurate measurement. The results indicate that if $R_o > 50$, the time variation of θ_v becomes independent of the thermal boundary condition on $R = R_o$ (adiabatic or isothermal). This fact indicates that a very small amount of sample liquid is required in the present method.

Figure 2 shows the effects of aspect ratios $L(l/r_i)$ on the wire temperature θ_v. As the aspect ratio L decreases, the slope in the relation θ_v vs log Fo, decreases because the conductive heat loss to the lead wire takes up a large fraction of the heat generated in the hot wire. The effect of the insulating layer depth δ is also examined taking a platinum wire with $L = 200$ coated by Al_2O_3 film as an example. Though the results are not shown here, with an increase in δ, the temperature rise becomes smaller but the slope remains constant. This characteristic allows us to determine the thickness of the insulating layer, for example the Al_2O_3 layer, of the probe.

The effects of the parameters R_{c1} and R_{d1}. are shown in Fig. 3 for combinations of platinum wire ($L = 200$) and some liquids. As R_{c1}

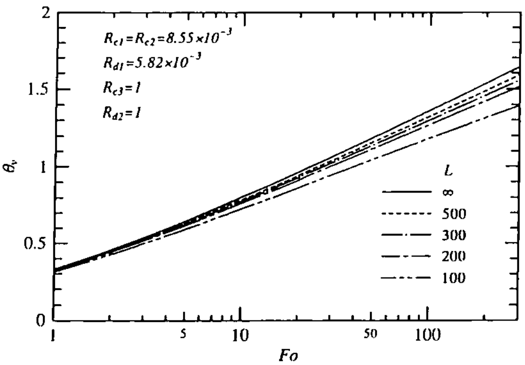


Fig. 2. Effects of aspect ratios L on θ_v.

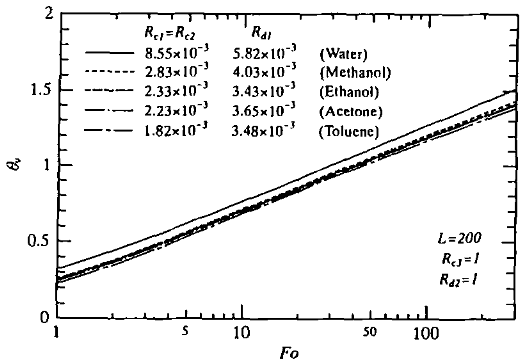


Fig. 3. Effects of parameters R_{c1} and R_{d1} on θ_v.

decreases, θ_v reduces because the heat loss from the wire edges to the supports increases relatively with decreases in the liquid thermal conductivity. Since the relation between θ_v and log Fo depends on the liquid thermal properties, numerical calculations with a wide range of parameters must be done before applying this method to unknown samples. This is the greatest difference from the conventional hot-wire method, which stands on a single working equation.

3. PRINCIPLE OF MEASUREMENTS

Based on the numerical results, the following procedure is proposed to determine simultaneously the thermal conductivity and thermal diffusivity of a liquid. As shown in Fig. 3, the dimensionless temperature increases linearly from $\mathrm{Fo}=5$ to at least $\mathrm{Fo}=150$. Therefore in this range of time, we regard the effects of wire heat capacity and heat loss from the wire edge as negligible. The real time corresponding to $\mathrm{Fo}=150$ ranges from 0.5 to 2 s, depending on the kind of liquid. Then the numerical results in this time range are approximated by a linear equation, Eq. (6), and coefficients A and B are determined by the least-squares method.

$$\theta_v = A \ln \mathrm{Fo} + B \tag{6}$$

The measured temperature rise of a wire with the same aspect ratio L is also approximated by a linear equation with coefficients a and b in the above time range as

$$T_v = a \ln t + b \tag{7}$$

where T_v is the volume-averaged wire temperature. Equation (6) is dimensionalized as

$$T_v = \frac{q_v r_i^2}{\lambda} A \ln t + \frac{q_v r_i^2}{\lambda}\left(A \ln \frac{\alpha}{r_i^2} + B\right) \tag{8}$$

Comparing the corresponding coefficients of Eqs. (7) and (8), the thermal conductivity and thermal diffusivity of a liquid are expressed as functions of these coefficients by

$$\lambda = q_v r_i^2 \frac{A}{a} = \frac{VI}{2\pi l}\frac{A}{a} \tag{9}$$

$$\alpha = r_i^2 \exp\left(\frac{b}{a} - \frac{B}{A}\right) \tag{10}$$

where V and I are the voltage and current supplied to the wire. Equations (9) and (10) are similar to those obtained for the conventional transient hot-wire method [3], except that $A$and B are changed a little with aspect ratio L, parameters R_{c1} and R_{d1}, etc., so that an iteration process is required to evaluate thermal properties accurately.

From Eqs. (9) and (10) the relative errors of the thermal conductivity and thermal diffusivity are estimated by

$$\frac{\delta\lambda}{\lambda} = \left\{\left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta a}{a}\right)^2\right\}^{1/2} \tag{11}$$

$$\frac{\delta\alpha}{\alpha} = \left[\left(\frac{2\delta r_i}{r_i}\right)^2 + \left\{\delta\left(\frac{B}{A}\right)\right\}^2 + \left\{\delta\left(\frac{b}{a}\right)\right\}^2\right]^{1/2} \tag{12}$$

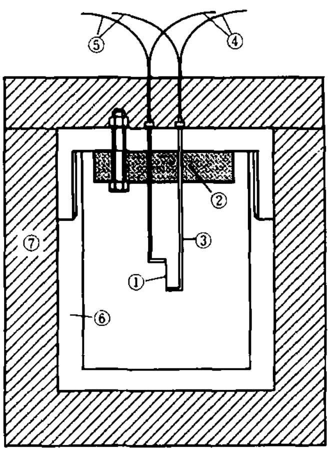
The magnitudes of the main factors in Eqs. (11) and (12) are estimated as follows. The length and radius of the hot wire are measured with a microcathetometer and microscope, and both $\delta l/l$ and $\delta r_i/r_i$ are accurate to 1%. The possible error in the slope of the temperature against ln t includes the uncertainties induced by electrical noise and the timing of the voltage measurements. The maximum deviation of the temperature measurement from Eq. (7) is less than 0.2%. The values of $\delta a/a$ and $\delta(b/a)$ are 0.01 and 0.04, respectively. From numerical solutions, $\delta A/A$ is found to be 0.002 and $\delta(B/A)$ is 0.003. The voltage and current through the wire are measured with digital multimeters and the values of $\delta V/V$ and $\delta I/I$ in the measurement are less than 10^{-4}. The total errors of this method are estimated to be 2 and 5% for thermal conductivity and thermal diffusivity, respectively.

4. MEASUREMENTS

4.1. Hot-Wire Cell

Figure 4 shows the transient short-hot-wire cell used in this study. A short platinum wire, about 50 μm in diameter and 10 mm in length (1), is welded at both ends to a platinum lead wire 1 mm in diameter (3) which is supported with a ceramic circular plate (2) and connected with voltage (4) and current (5) lead wires. The ceramic plate is fixed to the lid of a cylindrical vessel (6) which is made of Teflon, having 50 mm in diameter and 70 mm in height, and covered by a thermal insulator (7). The measurements are made at room temperature and atmospheric pressure.

The platinum hot wire is annealed at 800°C for a few hours and the temperature coefficient of its electrical resistance β is determined by calibration in the temperature range 0–60°C. This wire is cut and welded to the platinum lead terminals. The probe is again calibrated to determine its total resistance including the lead terminals Rt_0 at 0°C, under the assumption that the coefficient of the lead wire remains at β. When the probe is



① Platinum wire (50μm ϕ)
② Ceramic plate (48 mm ϕ, 10 mm thick)
③ Platinum lead terminal (1 mm ϕ)
④ Voltage terminal
⑤ Current terminal
⑥ Test vessel (Teflon)
⑦ Insulating material

Fig. 4. Short-hot-wire cell.

immersed in a liquid with uniform temperature, the hot-wire temperature is obtained as

$$T_v = \frac{1}{\beta}\left(\frac{Ri}{Rt_0} - 1\right) \tag{13}$$

where Ri is the measured resistance of the probe. On the other hand, when the probe is heated, the wire temperature rises but the lead terminal temperature remains at the initial temperature because of its large heat capacity. Therefore, the hot wire's temperature is estimated as

$$T_v = \frac{1}{\beta}\left(\frac{Rt(t) - \varepsilon Ri}{(1-\varepsilon)\, Rt_0} - 1\right) \tag{14}$$

where ε is the resistance ratio of the lead terminals and the entire probe and is 0.027–0.030 for the present probes.

4.2. Data Acquisition

The measuring system is schematically shown in Fig. 5. It is composed of a DC power supply and a voltage and current measuring and control system, that is, a Digital Multimeter PC and PI/O controller. The power supply can generate a constant current 1 A at maximum with 1.5-mA resolution. Two DMs are the same type and have a 6.5-figure accuracy at a sampling rate of 30 per s. The controller with PC handles both switching and logging of data.

The data acquisition procedure is almost the same as for the conventional hot-wire method. After the liquid temperature becomes uniform and

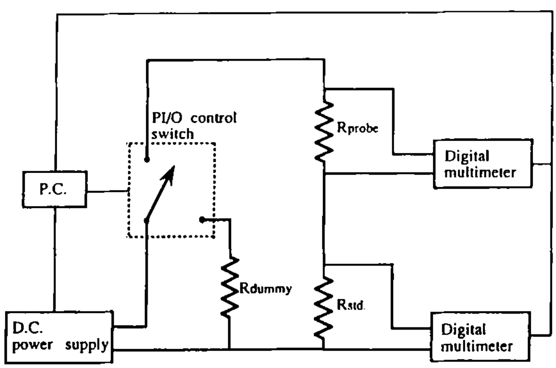


Fig. 5. Schematic of measuring system.

constant, a small current, about 3 mA, is supplied to the probe for 3 s to measure the initial liquid temperature. Then the switch is turned on to the dummy circuit, having the same resistance as the main circuit including the probe, and a heating current of 0.1–0.3 A is supplied. After 1 s, when the current becomes stable, the switch is again closed to the main circuit to begin heating the hot wire. In this process, the current and voltage are measured 30 times per s. These measurements are carried out automatically using a sequential program and GP-IB controlled by a personal computer. The measuring system was installed into a capsule of JAMIC (Japan Microgravity Center) and experienced measurements under microgravity conditions during about 10 s of its free fall in the drop shaft.

5. RESULTS AND DISCUSSION

First, the characteristics of the short-hot-wire probe are examined by measuring pure water as a standard reference material for liquid thermal conductivity. The temperature rise is compared with numerical results, and the evaluated thermal conductivity and thermal diffusivity are compared with reference values. Then the effective hot-wire length and diameter are determined. The length differs at most 3% from that measured with a microcathetometer. The reason for the difference is attributed to bending of the wire and to uncertainty of accurate welding positions on the lead terminals. In the present measurements, the effective length corrected from the pure-water measurement is used as the probe length. Since the present paper treats only electrically nonconducting liquids, the probe has no insulating layer, that is, a bare platinum wire.

The thermal conductivity and thermal diffusivity of five liquids have been measured, that is, ethanol, methanol, toluene, acetone, and pure water as a standard liquid, as mentioned above. The measurements have been made under microgravity conditions as well as normal gravity conditions. The reproducibility of the hot-wire temperature rise is examined for water and it is confirmed that the differences among the data obtained repeatedly are within 0.01°C, if the interval between successive measurements is more than 30 min.

Figure 6 shows examples of the time evolution of the temperature rise. Open and filled symbols indicate data taken under normal gravity and microgravity conditions, respectively. The volume-averaged wire temperature increases linearly during the whole heating period under microgravity conditions. On the other hand, under the normal gravity condition, the temperature also increases linearly until a certain critical time, then deviates from the linear line. After showing a maximum value, it tends

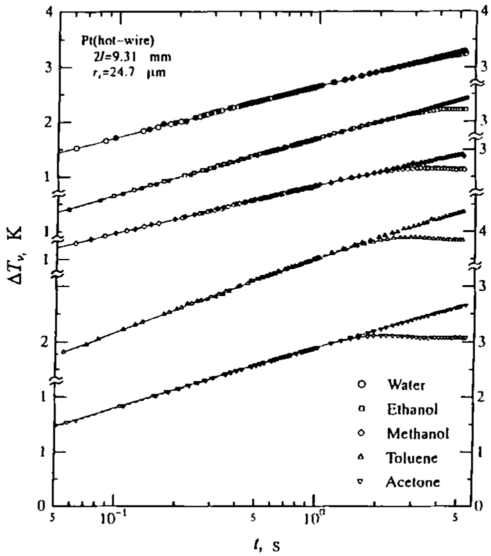


Fig. 6. Measured temperature evolution with time.

to a steady state. This is attributed to the effects of natural convection flow induced along the wire.

The critical time, at which the linear relation breaks, varies from 1 to 4 s depending on the liquid. The dimensionless critical time Fo_c is plotted for trial against the dimensionless parameter $Ra_l^{*\,2/3}/L^3$ in Fig. 7, where the Rayleigh number Ra_l^* $(=Ra_l Nu_l)$ is defined by the wire length and the measured temperature difference at the critical time. In Fig. 7, the three dashed lines indicate the relationship between the critical time and the heat flux and thermal properties of fluid suggested by Pantaloni et al. [4], which are rearranged by the present parameter $Ra_l^{*\,2/3}/L^3$. Their results are changed with different aspect ratios because the effect of the aspect ratio is not considered in their paper. On the other hand, the data obtained by Ro et al. [5] for 100-mm-long wires with different diameters as well as those of our present measurements for a 10-mm-long wire are plotted along a single straight line, the solid line. These data are correlated by

$$\mathrm{Fo}_c = 8.3 Ra_l^{*\,-2/3} L^3 \tag{15}$$

As shown in Fig. 7, Fo_c decreases monotonically with an increase in $Ra_l^{*\,2/3}/L^3$, which means that the critical time decreases with two-thirds of the heat flux and increases with one-third of the wire-aspect ratio. The above equation could be a general relation to determine the critical time

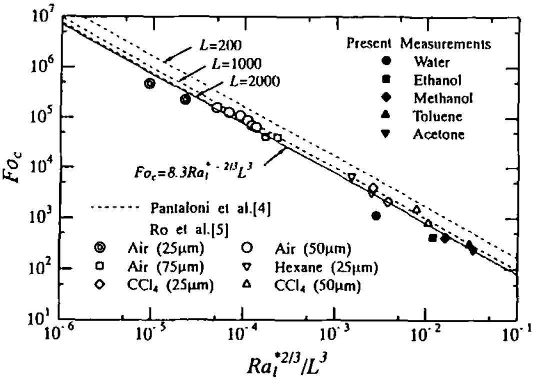


Fig. 7. Critical time for onset of natural convection.

for the transient hot-wire method. It is obvious that even for a liquid having large volumetric expansion coefficients such as acetone, the effect of natural convection will not appear, at least in the range $Fo < 200$.

From Eqs. (9) and (10) the thermal conductivity and thermal diffusivity of these liquids are obtained simultaneously. Table I compares values thus obtained with relevant reference values [6–8]. The values in the upper and lower rows for each liquid are obtained under normal and microgravity conditions, respectively. The former value for each liquid is a value averaged over several measurements. Because the probe characteristics are determined from the measurement of pure water, the relative errors are very small for water. For other liquids, the measured values of

Table I. Measured Thermal Conductivity and Thermal Diffusivity

Substance	Temperature T (°C)	Ref. data α ($m^2 \cdot s^{-1}$)	Measured α ($m^2 \cdot s^{-1}$)	Ref. data λ ($W \cdot m^{-1} \cdot K^{-1}$)	Measured λ ($W \cdot m^{-1} \cdot K^{-1}$)	Relative errors of α and λ (%)
Water	22.25	1.444E−7	1.45E−7	0.6016	0.605	0.4, 0.6
	25.17	1.458E−7	1.43E−7	0.6067	0.608	−1.9, 0.2
Methanol	20.91	1.030E−7	1.03E−7	0.2039	0.194	0, −4.9
	26.01	1.017E−7	1.02E−7	0.2024	0.198	0.3, −2.2
Ethanol	24.56	8.723E−8	9.00E−8	0.1667	0.165	3.2, −1.0
	23.70	8.753E−8	8.75E−8	0.1669	0.165	0, −1.1
Acetone	19.94	9.325E−8	9.69E−8	0.1620	0.158	3.9, −2.5
	26.04	9.206E−8	9.19E−8	0.1597	0.156	−0.2, −2.3
Toluene	22.78	8.858E−8	9.08E−8	0.1317	0.129	2.5, −2.1
	25.91	8.783E−8	8.80E−8	0.1308	0.131	0.2, 0.2

the thermal conductivity and thermal diffusivity are close to the reference values within the respective error estimated previously, except for the thermal conductivity of methanol, which is slightly lower than the reference value. But the value is very close to the reference value of Reid et al. [9] and remains still within the estimated error. Although the differences in the measured values under normal vs microgravity conditions are not remarkable, the microgravity measurement seems to provide slightly more accurate results.

6. CONCLUSIONS

The main conclusions are as follows.

(a) A new method has been proposed to measure the thermal conductivity and thermal diffusivity of a liquid simultaneously. The method uses a short-hot-wire probe 10 mm in length and 50 μm in diameter. The method can yield the thermal conductivity and thermal diffusivity within 2 and 5%, respectively.

(b) The measured thermal conductivity and thermal diffusivity of five liquids under both normal and microgravity conditions agree well with the reference values. From a comparison of these results, the critical time for onset of natural convection is clarified and a general relation with respect to the time is proposed.

(c) Though the accuracy is not as high as for the conventional transient hot-wire measurement, the present method is much more convenient and has a sufficient accuracy to yield approximate values for new liquids. The method, therefore, will be applied for measuring these properties of electrically conducting liquids like molten salt, when the probe is adequately coated with insulating and anticorrosive materials.

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